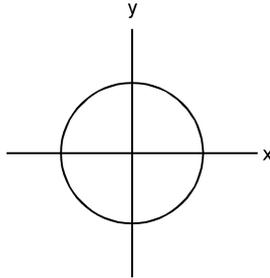


# EXERCISE-1

## SECTION (A) : KINEMATICS OF CIRCULAR MOTION

- A 1. Figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is  $v = (2\text{ m/s}) \hat{i} - (2\text{ m/s}) \hat{j}$ . Through which quadrant is the particle moving when it is travelling (a) clockwise and (b) counter clockwise around the circle?



## SECTION (B) : RADIAL AND TANGENTIAL ACCELERATION

- B 1. A car is moving with speed 30 m/sec on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/sec<sup>2</sup>. What is the acceleration of the car at that moment?

## SECTION (C) : RADIUS OF CURVATURE AND DYNAMICS OF CIRCULAR MOTION

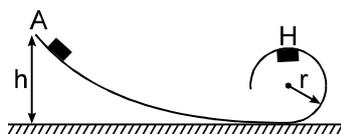
- C 1. A mass is kept on a horizontal frictionless surface. It is attached to a string and rotates about a fixed centre at an angular velocity  $\omega_0$ . If the length of the string and angular velocity are doubled, find the tension in the string which was initially  $T_0$ .
- C 2. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical.
- C 3. What is the radius of curvature of the parabola traced out by the projectile in the previous problem in the previous problem at a point where the particle velocity makes an angle  $\theta/2$  with the horizontal?
- C 4. A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1500 at full speed. Consider a particle of mass 1g sticking at the outer end of a blade. How much force does it experience when the fan runs at full speed? Who exerts this force on the particle? How much force does the particle exert on the blade along its surface?

## SECTION (D) : CIRCULAR MOTION IN HORIZONTAL PLANE

- D 1. A string breaks if its tension exceeds 10 newtons. A stone of mass 250 gm tied to this string of length 10 cm is rotated in a horizontal circle. Find the maximum angular velocity of rotation.
- D 2. A body of mass  $m$  hangs at one end of a string of length  $a$ , the other end of which is fixed. It is given a horizontal velocity  $u$  at its lowest position so that the string would just become slack, when it makes an angle of  $60^\circ$  with the upward drawn vertical line. Find the tension in the string at point of projection.
- D 3. A ball is moving to and fro about the lowest point A of a smooth hemispherical bowl. If it is able to rise up to a height of 20 cm on either side of A, find its speed at A. (Take  $g = 10\text{ m/s}^2$ , mass of the body 5 g).
- D 4. A motorcyclist wants to drive on the vertical surface of wooden 'well' of radius 5 m, with a minimum speed of  $5\sqrt{5}\text{ m/s}$ . Find the minimum value of coefficient of friction between the tyres and the wall of the well. (take  $g = 10\text{ m/s}^2$ )

**SECTION (E) : CIRCULAR MOTION IN VERTICAL PLANE**

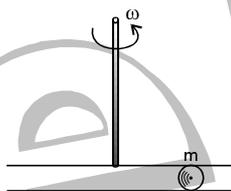
- E 1. A small body of mass  $m$  is allowed to slide on an inclined frictionless track from rest position as shown in the figure.



- (i) Find the minimum height  $h$ , so that body may successfully complete the loop of radius ' $r$ '.  
 (ii) If  $h$  is double of that minimum height, find the resultant force on the block at position H
- E 2. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle of  $60^\circ$  from the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolution with the nail as centre. Assume the length of pendulum to be 1 m.
- E 3. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill, when the bucket is at the highest position (Take  $g = 10 \text{ m/sec}^2$ )

**SECTION (F) : MOTION OF A VEHICLE, CENTRIFUGAL FORCE AND ROTATION OF EARTH**

- F 1. A train has to negotiate a curve of radius 400 m. By how much height should the outer rail be raised with respect to inner rail for a speed of 48 km/hr ? The distance between the rails is 1 m :
- F 2. A particle of mass  $m$  is constrained to move along a groove which is being rotated about a vertical axis through its centre with a constant angular velocity  $\omega$ . If it starts at a distance  $R$  from the axis at  $t = 0$ , find its velocity relative to the groove when it is at a distance  $r$  from the centre



- F 3. A park has a radius of 10m. If a vehicle goes round it at an average speed of 18 km/hr , what should be the proper angle of banking?
- F 4. If the road of the previous problem is horizontal (no banking) , what should be the minimum friction coefficient so that a scotter going at 18 km/hr does not skid.
- F 5. A person stands on a spring balance at the equator. (A) By what fraction is the balance reading less than his true weight ? (B) If the speed of earth's rotation is increased by such an amount that the balance reading is half the true weight, what will be the length of the day in this case ?
- F 6. A turn of radius 20 m is banked for the vehicles going at a speed of 36 km/h. If the coefficient of static friction between the road and the tyre is 0.4, what are the possible speeds of a vehicle so that is neither slips down nor skids up ?

**EXERCISE-2**

**SECTION (A) : KINEMATICS OF CIRCULAR MOTION**

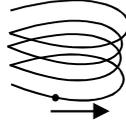
- A 1. The average acceleration vector for a particle having a uniform circular motion in one complete revolution:
- (A) A constant vector of magnitude  $\frac{v^2}{r}$
- (B)  $\frac{v^2}{r}$  in magnitude and perpendicular to the plane of circle
- (C) Equal to the instantaneous acceleration vector at the start of the motion
- (D) A null vector

- A 2.** A particle moves along a circle of radius  $\left(\frac{20}{\pi}\right)$  m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is:  
 (A)  $160 \pi \text{ m/s}^2$  (B)  $40 \pi \text{ m/s}^2$  (C)  $40 \text{ m/s}^2$  (D)  $640 \pi \text{ m/s}^2$
- A 3.** When a particle moves in a circle with a uniform speed  
 (A) its velocity and acceleration are both constant  
 (B) its velocity is constant but the acceleration changes  
 (C) its acceleration is constant but the velocity changes  
 (D) its velocity and acceleration both change
- A 4.** An object follows a curved path. The following quantities may remain constant during the motion  
 (A) speed (B) velocity (C) acceleration (D) magnitude of acceleration
- A 5.** Assume that the earth goes round the sun in a circular orbit with a constant speed of 30 km/s.  
 (A) The average velocity of the earth from 1st Jan, 90 to 30th June, 90 is zero  
 (B) The average acceleration during the above period is  $60 \text{ km/s}^2$ .  
 (C) The average speed from 1st Jan, 90 to 31st Dec, 90 is zero.  
 (D) The instantaneous acceleration of the earth points towards the sun.
- A 6.** The position vector of a particle in a circular motion about the origin sweeps out equal area in equal time. Its  
 (A) velocity remains constant (B) speed remains constant  
 (C) acceleration remains constant (D) tangential acceleration remains constant

**SECTION (B) : RADIAL AND TANGENTIAL ACCELERATION**

- B 1.** Two particles P and Q are located at distances  $r_p$  and  $r_q$  respectively from the centre of a rotating disc such that  $r_p > r_q$  :  
 (A) Both P and Q have the same acceleration (B) Both P and Q do not have any acceleration  
 (C) P has greater acceleration than Q (D) Q has greater acceleration than P
- B 2.** In uniform circular motion, the quantity that remains constant is :  
 (A) linear velocity (B) centripetal force (C) acceleration (D) speed
- B 3.** When a particle moves in a circle with a uniform speed :  
 (A) Its velocity and acceleration are both constant  
 (B) Its velocity is constant but the acceleration changes  
 (C) Its acceleration is constant but the velocity changes  
 (D) Its velocity and acceleration both change
- B 4.** For a particle in a uniformly accelerated (speed increasing uniformly) circular motion :  
 (A) velocity is radial and acceleration is tangential only  
 (B) velocity is tangential and acceleration is radial only  
 (C) velocity is radial and acceleration has both radial and tangential components  
 (D) velocity is tangential and acceleration has both radial and tangential components
- B 5.** A motor cyclist going round in a circular track at constant speed has  
 (A) constant linear velocity (B) constant acceleration  
 (C) constant angular velocity (D) constant force
- B 6.** For a body in circular motion with a constant angular velocity, the magnitude of the average acceleration over a period of half a revolution is .....times the magnitude of its instantaneous acceleration.  
 (A)  $\frac{2}{\pi}$  (B)  $\frac{\pi}{2}$  (C)  $\pi$  (D) 2
- B 7.** A car of mass  $m$  moves in a horizontal circular path of radius  $r$  metre. At an instant its speed is  $v \text{ m/s}$  and is increasing at a rate  $a \text{ m/s}^2$ , then the acceleration of the car is :  
 (A)  $\sqrt{a\left(\frac{v^2}{r}\right)}$  (B)  $\sqrt{a^2 + \left(\frac{v^2}{r}\right)^2}$  (C)  $\frac{v^2}{r}$  (D)  $a$

- B 8.** The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be :  
 (A) 6.28 & 0 mm/s (B) 8.88 & 4.44 mm/s  
 (C) 8.88 & 6.28 mm/s (D) 6.28 & 8.88 mm/s
- B 9.** A particle is going in a spiral path as shown in figure with constant speed.



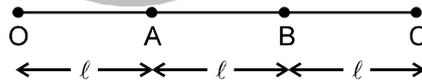
- (A) The velocity of the particle is constant  
 (B) The acceleration of the particle is constant  
 (C) The magnitude of acceleration is constant  
 (D) The magnitude of acceleration is decreasing continuously.

**SECTION (C) : RADIUS OF CURVATURE AND DYNAMICS OF CIRCULAR MOTION**

- C 1.** A particle of mass  $m$  is executing uniform circular motion on a path of radius  $r$ . If  $p$  is the magnitude of its linear momentum. The radial force acting on the particle is :  
 (A)  $p m r$  (B)  $\frac{r m}{p}$  (C)  $\frac{m p^2}{r}$  (D)  $\frac{p^2}{r m}$
- C 2.** A train is moving towards north. At one place it turns towards north-east, here we observe that  
 (A) the radius of curvature of outer rail will be greater than that of the inner rail  
 (B) the radius of the inner rail will be greater than that of the outer rail  
 (C) the radius of curvature of one of the rails will be greater  
 (D) The radius of curvature of the outer and inner rails will be the same

**SECTION (D) : CIRCULAR MOTION IN HORIZONTAL PLANE**

- D 1.** A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16 newton. The maximum speed of revolution of the stone without breaking it, will be :  
 (A) 20  $\text{ms}^{-1}$  (B) 16  $\text{ms}^{-1}$  (C) 14  $\text{ms}^{-1}$  (D) 12  $\text{ms}^{-1}$
- D 2.** Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point O. If the velocity of the outermost particle is  $v_0$ , then the ratio of tensions in the three sections of the string is :



- (A) 3 : 5 : 7 (B) 3 : 4 : 5 (C) 7 : 11 : 6 (D) 3 : 5 : 6
- D 3.** A particle is kept fixed on a turntable rotating uniformly. As seen from the ground , the particle goes in a circle , its speed is 20 cm/s and acceleration is 20  $\text{cm/s}^2$ . The particle is now shifted to a new position to make the radius half of the original value. The new values of the speed and acceleration will be  
 (A) 10 cm/s, 10  $\text{cm/s}^2$  (B) 10 cm/s, 80  $\text{cm/s}^2$  (C) 40 cm/s, 10  $\text{cm/s}^2$  (D) 40 cm/s, 40  $\text{cm/s}^2$
- D 4.** A coin placed on a rotating turntable just slips if is placed at a distance of 4 cm from the centre. if the angular velocity of the turntable is doubled , it will just slip at a distance of  
 (A) 1 cm (B) 2 cm (C) 4 cm (D) 8 cm
- D 5.** A rod of length  $L$  is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane . Let  $T_1$  and  $T_2$  be the tensions at the points  $L/4$  and  $3L/4$  away from the pivoted ends.  
 (A)  $T_1 > T_2$  (B)  $T_2 > T_1$  (C)  $T_1 = T_2$   
 (D) The relation between  $T_1$  and  $T_2$  depends on whether the rod rotates clockwise or anticlockwise

**SECTION (E) : CIRCULAR MOTION IN VERTICAL PLANE**

- E 1.** When a particle is rotated in a vertical plane with constant angular velocity, magnitude of centripetal force is :

- (A) Maximum at highest point (B) Maximum at lowest point  
 (C) Same at all points (D) zero
- E 2. A motorcycle is going on an overbridge of radius R. The driver maintains a constant speed. As the motorcycle is ascending on the overbridge, the normal force on it :  
 (A) increase (B) decreases  
 (C) remains constant (D) first increases then decreases.

- E 3. A heavy particle hanging vertically from a point by a light inextensible string of length  $l$  is started so as to make a complete revolution in a vertical plane. The sum of the magnitude of tension at the ends of any diameter :  
 (A) first increase then decreases (B) is constant  
 (C) first decrease then increases (D) decreases continuously

- E 4. In a circus stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be :  
 (A)  $\sqrt{2gR}$  (B)  $2gR$  (C)  $\sqrt{3gR}$  (D)  $\sqrt{gR}$

- E 5. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break  
 (A) when the mass is at the highest point of the circle  
 (B) when the mass is at the lowest point of the circle  
 (C) when the wire is horizontal  
 (D) at an angle of  $\cos^{-1}(1/3)$  from the upward vertical

- E 6. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles  $30^\circ$  and  $60^\circ$  from vertical (lowest positions) are  $T_1$  and  $T_2$  respectively. Then  
 (A)  $T_1 = T_2$  (B)  $T_2 > T_1$   
 (C)  $T_1 > T_2$  (D) Tension in the string always remains the same

- E 7. A car moves at a constant speed on a road as shown in figure. The normal force by the road on the car in  $N_A$  and  $N_B$  when when it is at the points A and B respectively.



- (A)  $N_A = N_B$  (B)  $N_A > N_B$  (C)  $N_A < N_B$  (D) insufficient

- E 8. Water in a bucket is whirled in a vertical circle with a string attached to it. The water does not fall down even when the bucket is inverted at the top of its path. We conclude that in this position.

- (A)  $mg = \frac{mv^2}{r}$  (B)  $mg$  is greater than  $\frac{mv^2}{r}$   
 (C)  $mg$  is not greater than  $\frac{mv^2}{r}$  (D)  $mg$  is not less than  $\frac{mv^2}{r}$

- E 9. A simple pendulum having a bob of mass  $m$  is suspended from the ceiling of a car used in a stunt film shooting. The car moves up along an inclined cliff at a speed  $v$  and makes a jump to leave the cliff and lands at some distance. Let  $R$  be the maximum height of the car from the top of the cliff. The tension in the string when the car is in air is

- (A)  $mg$  (B)  $mg - \frac{mv^2}{r}$  (C)  $mg + \frac{mv^2}{r}$  (D) zero

**SECTION (F) : MOTION OF A VEHICLE, CENTRIFUGAL FORCE AND ROTATION OF EARTH**

- F 1. The driver of a car travelling at speed suddenly sees a wall at a distance  $r$  directly in front of him. To avoid collision,  
 (A) he should apply brakes sharply (B) he should turn the car sharply  
 (C) he should apply brakes and then sharply turn (D) None of these

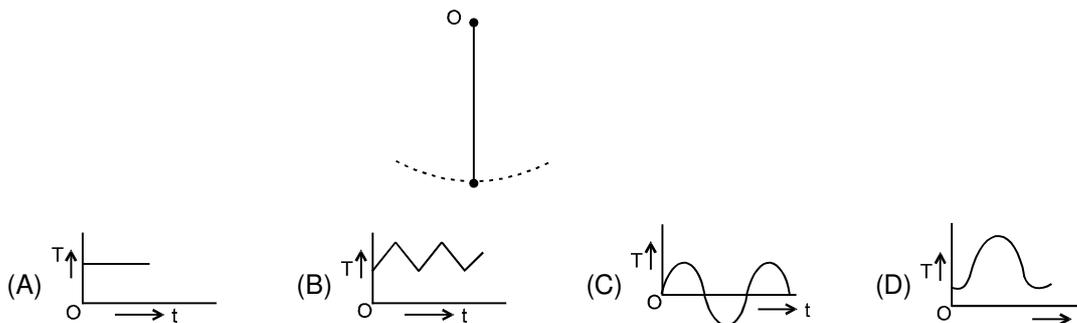
- F 2.** A curved section of a road is banked for a speed  $v$ . If there is no friction between road and tyres of the car, then:  
 (A) car is more likely to slip at speeds higher than  $v$  than speeds lower than  $v$   
 (B) car cannot remain in static equilibrium on the curved section  
 (C) car will not slip when moving with speed  $v$   
 (D) none of the above
- F 3.** A particle of mass  $m$  is observed from an inertial frame of reference and is found to move in a circle of radius  $r$  with a uniform speed  $v$ . The centrifugal force on it is  
 (A)  $\frac{mv^2}{r}$  towards the centre (B)  $\frac{mv^2}{r}$  away from the centre  
 (C)  $\frac{mv^2}{r}$  along the tangent through the particle (D) zero
- F 4.** A train A runs from east to west and another train B of the same mass runs from west to east at the same speed along the equator. A presses the track with a force  $F_1$  and B presses the track with a force  $F_2$ .  
 (A)  $F_1 > F_2$   
 (B)  $F_1 < F_2$   
 (C)  $F_1 = F_2$   
 (D) the information is insufficient to find the relation between  $F_1$  and  $F_2$ .
- F 5.** A car of mass  $M$  is moving horizontally on a circular path of radius  $r$ . At an instant its speed is  $v$  and is increasing at a rate  $a$ .  
 (A) The acceleration of the car is towards the centre of the path  
 (B\*) The magnitude of the frictional force on the car is greater than  $\frac{mv^2}{r}$   
 (C\*) The friction coefficient between the ground and the car is not less than  $a/g$ .  
 (D) The friction coefficient between the ground and the car is  $\mu = \tan^{-1} \frac{v^2}{rg}$
- F 6.** A circular road of radius  $r$  is banked for a speed  $v = 40$  km/hr. A car of mass  $m$  attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible.  
 (A) The car cannot make a turn without skidding.  
 (B\*) If the car turns at a speed less than 40 km/hr, it will slip down  
 (C) If the car turns at the current speed of 40 km/hr, the force by the road on the car is equal to  $\frac{mv^2}{r}$   
 (D\*) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is greater than  $mg$  as well as greater than  $\frac{mv^2}{r}$
- F 7.** A person applies a constant force  $\vec{F}$  on a particle of mass  $m$  and finds that the particle moves in a circle of radius  $r$  with a uniform speed  $v$  as seen from an inertial frame of reference.  
 (A) This is not possible.  
 (B) There are other forces on the particle  
 (C) The resultant of the other forces is  $\frac{mv^2}{r}$  towards the centre.  
 (D) The resultant of the other forces varies in magnitude as well as in direction.

## EXERCISE-3

1. A particle of mass  $m$  begins to slide down a fixed smooth sphere from the top. What is its tangential acceleration when it breaks off the sphere ?  
 (A)  $\frac{2g}{3}$  (B)  $\frac{\sqrt{5}g}{3}$  (C)  $g$  (D)  $\frac{g}{3}$

2. A 1 kg stone at the end of 1 m long string is whirled in a vertical circle at constant speed of 4 m/sec. The tension in the string is 6 N when the stone is at ( $g = 10 \text{ m/sec}^2$ ):  
 (A) top of the circle (B) bottom of the circle (C) halfway down (D) none of these
3. A boy whirls a stone in a horizontal circle 1.8 m above the ground by means of a string 1.2 m long. The string breaks and stone flies off horizontally, striking the ground 9.1 m away. The centripetal acceleration during the circular motion was: (use  $g = 9.8 \text{ m/s}^2$ )  
 (A)  $94 \text{ m/s}^2$  (B)  $141 \text{ m/s}^2$  (C)  $188 \text{ m/s}^2$  (D)  $282 \text{ m/s}^2$

4. A particle of mass  $m$  is suspended from a fixed point O by a string of length  $\ell$ . At  $t = 0$ , it is displaced from equilibrium position and released. The graph, which shows the variation of the tension  $T$  in the string with time ' $t$ ', may be :



5. A particle P is moving in a circle of radius  $a$  with a uniform speed  $u$ . C is the centre of the circle and AB is diameter. The angular velocity of P about A and C are in the ratio :  
 (A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 4 : 2

6. The distance of the two planets from the sun are nearly  $10^{12}$  to  $10^{13}$  metres respectively. Then the ratio of their periodic time, if they are supposed to move in a circular orbit is :  
 (A)  $\sqrt{10} : 2$  (B)  $2 : \sqrt{10}$  (C)  $10\sqrt{10} : 1$  (D)  $1 : 10\sqrt{10}$

7. A stone of mass 1 kg tied to a light inextensible string of length  $L = \frac{10}{3} \text{ m}$ , whirling in a circular path in a vertical plane. The ratio of maximum tension in the string to the minimum tension in the string is 4, if  $g$  is taken to be  $10 \text{ m/s}^2$ , the speed of the stone at the highest point of the circle is :  
 (A) 10 m/s (B)  $5\sqrt{2} \text{ m/s}$  (C)  $10\sqrt{3} \text{ m/s}$  (D) 20 m/s

8. Toy cart tied of the end of an unstretched string of length  $a$ , when revolved moves in a horizontal circle of radius  $2a$  with a time period  $T$ . Now the toy cart is speeded up until it moves in a horizontal circle of radius  $3a$  with a period  $T'$ . If Hook's law holds then :  
 (A)  $T' = \sqrt{\frac{3}{2}} T$  (B)  $T' = \left(\frac{\sqrt{3}}{2}\right) T$  (C)  $T' = \left(\frac{3}{2}\right) T$  (D)  $T' = T$

9. A small sphere of mass  $m$  suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released, then :  
 (i) The total acceleration of the sphere and the thread tension as a function of  $\theta$ , the angle of deflection of the thread from the vertical will be  
 (A\*)  $g\sqrt{1+3\cos^2\theta}$ ,  $T = 3mg \cos \theta$  (B)  $g \cos \theta$ ,  $T = 3mg \cos \theta$ .  
 (C)  $g\sqrt{1+3\sin^2\theta}$ ,  $T = 5mg \cos \theta$  (D)  $g \sin \theta$ ,  $T = 5mg \cos \theta$ .  
 (ii) The thread tension at the moment when the vertical component of the sphere's velocity is maximum will be  
 (A)  $mg$  (B)  $mg\sqrt{2}$  (C\*)  $mg\sqrt{3}$  (D)  $\frac{mg}{\sqrt{3}}$   
 (iii) The angle  $\theta$  between the thread and the vertical at the moment when the total acceleration

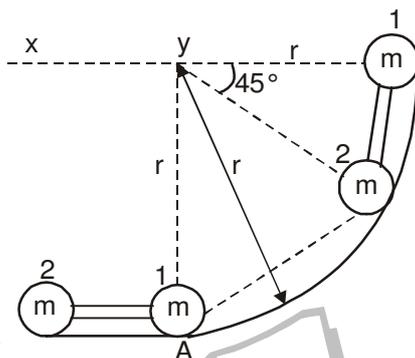
vector of the sphere is directed horizontally will be

- (A)  $\cos \theta = \frac{1}{\sqrt{3}}$       (B)  $\cos \theta = \frac{1}{3}$       (C)  $\sin \theta = \frac{1}{\sqrt{3}}$       (D)  $\sin \theta = \frac{1}{\sqrt{2}}$

10. The kinetic energy  $k$  of a particle moving along a circle of radius  $R$  depends on the distance covered  $s$  as  $k = as^2$  where  $a$  is a constant. The total force acting on the particle is :

- (A)  $2a \frac{s^2}{R}$       (B)  $2as \left(1 + \frac{s^2}{R^2}\right)^{1/2}$       (C)  $2as$       (D)  $2a \frac{R^2}{s}$

11. Two small spheres, each of mass  $m$  are rigidly connected by a rod of negligible mass and are released from rest in the position shown and slide down a smooth circular guide in the vertical plane.



(i) Their common velocity as they reach the horizontal dashed position will be :

- (A)  $\sqrt{2gr - \frac{1}{\sqrt{2}}gr}$       (B)  $10\sqrt{2gr - \frac{1}{\sqrt{2}}gr}$       (C)  $\sqrt{gr}$       (D)  $\sqrt{gr}/2$

(ii) The force  $R$  between sphere 1 and the supporting surface at an instant just before the sphere reaches the bottom position A will be :

- (A)  $22.9 \text{ mg}$       (B)  $2.29 \text{ mg}$       (C)  $mg$       (D)  $mg/2$

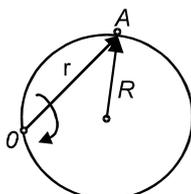
12. A particle moves with deceleration along the circle of radius  $R$  so that at any moment of time its tangential and normal accelerations are equal in moduli. At the initial moment  $t = 0$  the speed of the particle equals  $v_0$ , then :

- (i) the speed of the particle as a function of the distance covered  $s$  will be  
 (A)  $v = v_0 e^{-s/R}$       (B)  $v = v_0 e^{s/R}$       (C)  $v = v_0 e^{-R/s}$       (D)  $v = v_0 e^{R/s}$

(ii) the total acceleration of the particle as function of velocity and distance covered

- (A)  $a = \sqrt{2} \frac{v^2}{R}$       (B)  $a = \sqrt{2} \frac{v}{R}$       (C)  $a = \sqrt{2} \frac{R}{v}$       (D)  $a = \frac{2R}{v}$

13. A particle A moves along a circle of radius  $R = 50 \text{ cm}$  so that its radius vector  $r$  relative to the point O (Fig.) rotates with the constant angular velocity  $\omega = 0.40 \text{ rad/s}$ . Then modulus of the velocity of the particle, and the modulus of its total acceleration will be



- (A)  $v = 0.4 \text{ m/s}$ ,  $w = 0.4 \text{ m/s}^2$       (B)  $v = 0.32 \text{ m/s}$ ,  $w = 0.32 \text{ m/s}^2$   
 (C)  $v = 0.32 \text{ m/s}$ ,  $w = 0.4 \text{ m/s}^2$       (D)  $v = 0.4 \text{ m/s}$ ,  $w = 0.32 \text{ m/s}^2$

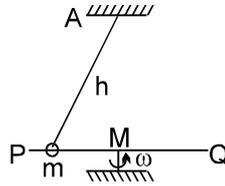
14. A spot light S rotates in a horizontal plane with a constant angular velocity of  $0.1 \text{ rad/s}$ . The spot of light P moves along the wall at a distance  $3 \text{ m}$ . What is the velocity of the spot P when  $\theta = 45^\circ$ ?

- (A)  $0.6 \text{ m/s}$       (B)  $0.5 \text{ m/s}$       (C)  $0.4 \text{ m/s}$       (D)  $0.3 \text{ m/s}$

# EXERCISE-4

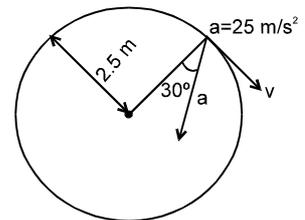
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1. A smooth rod PQ rotates in a horizontal plane about its mid point M which is  $h = 0.1$  m vertically below a fixed point A at a constant angular velocity  $14$  rad/s. A light elastic string of natural length  $0.1$  m requiring  $1.47$  N/cm has one end fixed at A and its other end attached to a ring of mass  $m = 0.3$  kg which is free to slide along the rod. When the ring is stationary relative to rod, then inclination of string with vertical, tension in string, force exerted by ring on the rod will be



- (A)  $\cos \theta = 3/5$ ,  $T = 9.8$  N,  $N = 2.88$  N                      (B)  $\theta = 60^\circ$ ,  $T = 0$ ,  $N = 1.44$  N  
 (C)  $\cos \theta = 2/5$ ,  $T = 4.9$  N,  $N = 1.44$  N                      (D)  $\theta = 30^\circ$ ,  $T = 0$ ,  $N = 2.88$  N

2. Figure shows the total acceleration and velocity of a particle moving clockwise in a circle of radius  $2.5$  m at a given instant of time. At this instant, find :



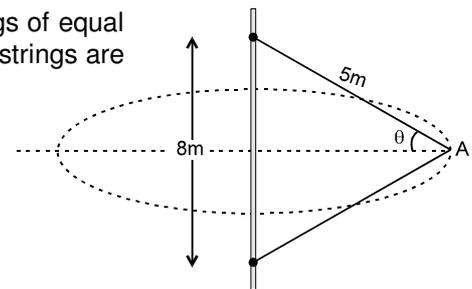
- (a) the radial acceleration,  
 (b) the speed of the particle and  
 (c) its tangential acceleration

3. Two particle A and B move anticlockwise with the same speed  $v$  in a circle of radius  $R$  and are diametrically opposite to each other. At  $t = 0$ , A is given a constant acceleration (tangential)  $a_t = \frac{72v^2}{25\pi R}$ . Calculate the time in which A collides with B, the angle traced by A, its angular velocity and radial acceleration at the time of collision.

4. A motorcycle has to move with a constant speed on an overbridge which is in the form of a circular arc of radius  $R$  and has a total length  $L$ . Suppose the motorcycle starts from the highest point.

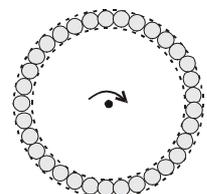
- (a) What can its maximum velocity be for which the contact with the road is not broken at the highest point.  
 (b) If the motorcycle goes at speed  $1/\sqrt{2}$  times the maximum found in part (a) where will it lose the contact with the road.  
 (c) What maximum uniform speed can it maintain on the bridge if it does not lose contact anywhere on the bridge?

5. A  $4$  kg block is attached to a vertical rod by means of two strings of equal length. When the system rotates about the axis of the rod, the strings are extended as shown in figure.



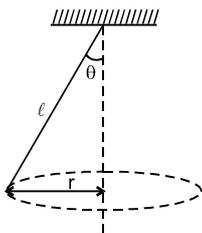
- (a) How many revolutions per minute must the system make in order for the tension in the upper chord to be  $20$  kgf?  
 (b) What is the tension in the lower chord then?

6. A metallic chain with a length  $\ell$  and whose ends are joined together is fitted onto a wooden disc as shown in the figure. The disc rotates with a speed of  $n$  revolutions per second. Find the tension of the chain  $T$  if its mass is  $m$ .



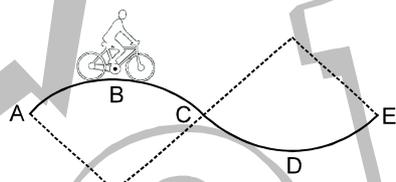
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7. A small object of mass  $m$  is tied to a string of length  $l$  and is whirled round in a horizontal circle of radius  $r$  at constant speed  $v$  (figure). The centre of the circle is vertically below the point of support. [This arrangement is called conical pendulum.]

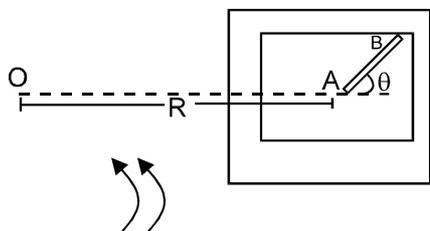


Calculate  $v$  in terms of  $g$ ,  $r$  and  $\theta$ . Also calculate the period of revolution.

8. A car goes on a horizontal circular road of radius  $R$ , the speed increasing at a constant rate  $\frac{dv}{dt} = a$ . The friction coefficient between the road and the tyre is  $\mu$ . Find the speed at which the car will skid.
9. A track consists of two circular parts  $ABC$  and  $CDE$  of equal radius  $100$  m and joined smoothly as shown in fig. Each part subtends a right angle at its centre. A cycle weighing  $100$  kg together with the rider travels at a constant speed of  $18$  km/h on the track. (A) Find the normal contact force by the road on the cycle when it is at  $B$  and  $D$ . (B) Find the force of friction exerted by the track on the tyres when the cycle is at  $B$ ,  $C$  and  $D$ . (c) Find the normal force between the road and the cycle just, before and just after the cycle crosses  $C$ . (D) What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed? Take  $g = 10\text{m/s}^2$ .

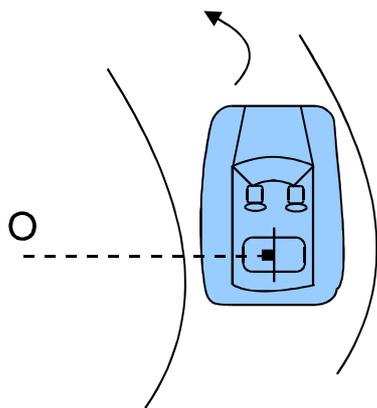


10. A block of mass ' $m$ ' moves on a horizontal circle against the wall of a cylindrical room of radius  $R$ . The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is  $\mu$ . The block is given an initial speed  $v_0$ . As a function of the instantaneous speed ' $v$ ' write  
 (A) the normal force by the wall on the block,  
 (B) the frictional force by the wall and  
 (C) the tangential acceleration of the block.  
 (D) obtain the speed of the block after one revolution.
11. A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity  $\omega$  in a circular path of radius  $R$  (figure). A smooth groove  $AB$  of length  $L (< R)$  is made on the surface of the table. The groove makes an angle  $\theta$  with the radius  $OA$  of the circle in which the cabin rotates. A small particle is kept at the point  $A$  in the groove and is released to move along  $AB$ . Find the time taken by the particle to reach the point  $B$ .

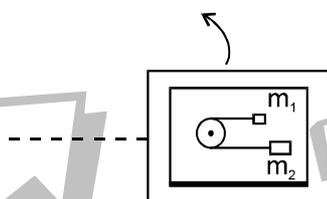


12. A car moving at a speed of  $36$  km/hr is taking a turn on a circular road of radius  $50$  m. A small wooden plate is kept on the seat with its plane perpendicular to the radius of the circular road figure. A small block of mass  $100\text{g}$  is kept on the seat which rests against the plate. The friction coefficient between the block and the plate is  $\mu = 0.58$ .  
 (A) Find the normal contact force exerted by the plate on the block.  
 (B) The plate is slowly turned so that the angle between the normal to the plate and radius of the angle

between the normal to the plate and the radius of the road slowly increases. Find the angle at which the block will just start sliding on the plate.



13. A table with smooth horizontal surface is placed in a cabin which moves in a circle of a large radius  $R$  (figure). A smooth pulley of small radius is fastened to the table. Two masses  $m$  and  $2m$  placed on the table are connected through a string over the pulley. Initially the masses are held by a person with the string along the outward radius and then the system is released from rest (with respect to the cabin). Find the magnitude of the initial acceleration of the masses as seen from the cabin and the tension in the string.



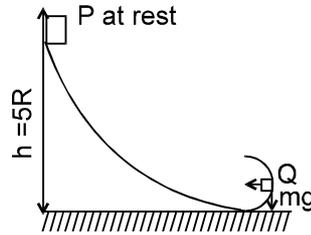
## EXERCISE-5

1. A hemispherical bowl of radius  $r = 0.1\text{ m}$  is rotating about its axis (which is vertical) with an angular velocity  $\omega$ . A particle of mass  $10^{-2}\text{ kg}$  on the frictionless inner surface of the bowl is also rotating with the same  $\omega$ . the particle is at a height  $h$  from the bottom of the bowl. (a) Obtain the relation between  $h$  and  $\omega$ . What is the minimum value of  $\omega$  needed in order to have a nonzero value of  $h$ . (b) It is desired to measure 'g' using this setup by measuring  $h$  accurately. Assuming that  $r$  and  $\omega$  are known precisely and that the least count in the measurement of  $h$  is  $10^{-4}\text{ m}$ . What is minimum error  $\Delta g$  in the measured value of  $g$ . [ $g = 9.8\text{ m/s}^2$ ]  
[JEE 1993]
2. A particle of mass  $m$  is moving in a circular path of constant radius  $r$  such that its centripetal acceleration  $a_c$  is varying with time  $t$  as  $a_c = k^2 r^2 t$  where  $k$  is a constant. The power delivered to the particle by the force acting on it is-  
[JEE 1994]  
(A)  $2\pi mk^2 r^2$       (B)  $mk^2 r^2 t$       (C)  $\frac{(mk^4 r^2 t^5)}{3}$       (D) Zero
3. A smooth semicircular wire track of radius  $R$  is fixed in a vertical plane (figure). On end of a massless spring of natural length  $3R/4$  is attached to the lowest point  $O$  of the wire track. A small ring of mass  $m$  which can slide on the track is attached to the other end of the spring. The ring is held stationary at point  $P$  such that the spring make an angle  $60^\circ$  with the vertical. The spring constant  $K = mg/R$ . Consider the instant when the ring is released  
(i) Draw the free body diagram of the ring.  
(ii) Determine the tangential acceleration of the ring and the normal reaction. [JEE 1996]
4. Two blocks of mass  $m_1 = 10\text{ Kg}$  and  $m_2 = 5\text{ Kg}$  connected to each other by a massless inextensible string of length  $0.3\text{ m}$  are placed along a diameter of turn table. The coefficient of friction between the table and  $m_1$  is  $0.5$  while there is no friction between  $m_2$  and the table. The table is rotating with an angular velocity of  $10\text{ rad/s}$  about a vertical axis passing through its centre  $O$ . The masses are placed along the diameter of the table on either side of the centre  $O$  such that the mass  $m_1$  is at a distance of  $0.124\text{ m}$  from  $O$ . The masses are observed to be at rest with respect to an observer on the turn table.  
(i) Calculate the frictional force on  $m_1$ .  
(ii) What should be the minimum angular speed of the turn table so that the masses will slip

- (iii) from this position.  
How should the masses be placed with the string remaining taut so that there is no frictional force acting on the mass  $m_1$ .

[JEE 1997]

5. A small block of mass  $m$  slides along a smooth frictional track as shown in figure. (i) If it starts from rest at P, when is the resultant force acting on it at Q? (ii) At what height above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight?



[REE 1997]

6. A stone tied to a string of length  $L$  is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed  $u$ . The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is

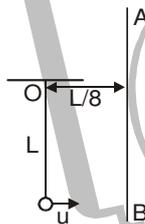
[JEE 1998]

- (A)  $\sqrt{u^2 - 2gL}$  (B)  $\sqrt{2gL}$  (C)  $\sqrt{u^2 - gL}$  (D)  $\sqrt{2(u^2 - gL)}$

7. A particle at rest starts rolling from the top of a large frictionless sphere of radius  $R$ . The sphere is fixed on the ground. Calculate that height from the ground at which the particle leaves the surface of the sphere.

[REE 1998]

8. A particle is suspended vertically from a point O by an inextensible massless string of length  $L$ . A vertical line AB is at a distance  $L/8$  from O as shown. The object given a horizontal velocity  $u$ . At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find  $u$ .



[JEE 1999]

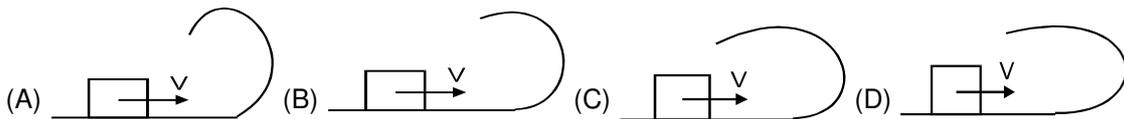
9. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance  $L$  from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration,  $\alpha$ . If the coefficient of friction between the rod and the bead is  $\mu$ , and gravity is neglected, then the time after which the bead starts slipping is -

[JEE 2000]

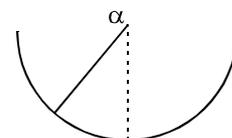
- (A)  $\sqrt{\frac{\mu}{\alpha}}$  (B)  $\frac{\mu}{\sqrt{\alpha}}$  (C)  $\frac{1}{\sqrt{\mu\alpha}}$  (D) Infinitesimal

10. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum is-

[JEE 2001]



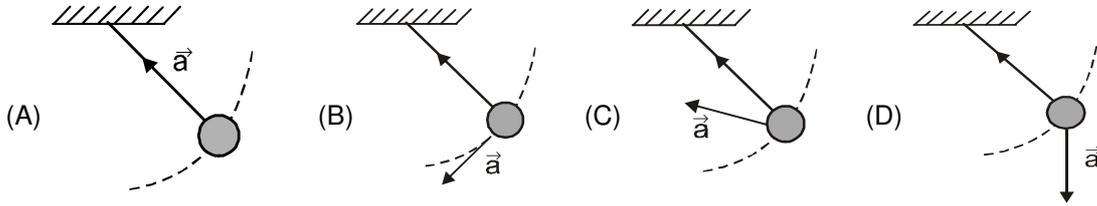
11. An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is  $1/3$ . If the line joining the centre of the hemispherical surface to the insect makes an angle  $\alpha$  with the vertical, the maximum possible value of  $\alpha$  is given-



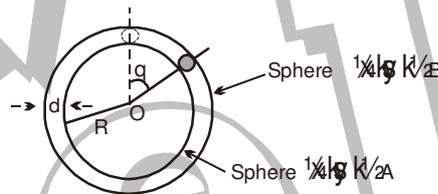
- (A)  $\cot \alpha = 3$  (B)  $\tan \alpha = 3$  (C)  $\sec \alpha = 3$  (D)  $\operatorname{cosec} \alpha = 3$  [JEE 2001]

12. A block is placed inside a horizontal hollow cylinder. The cylinder starts rotating with one revolution per second about its axis. The angular position of the block at which it begins to slide is  $30^\circ$  below the horizontal level passing through the center. Find the radius of the cylinder if the coefficient of friction is 0.6. What should be the minimum angular speed of the cylinder so that the block reach the highest point of the cylinder? [REE 2001]

13. A simple pendulum is oscillating without damping. When the displacements of the bob is less than maximum, its acceleration vector  $\vec{a}$  is correctly shown in [JEE 2002]



14. A spherical ball of mass  $m$  is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius  $R$  and the space between the two spheres has a width  $d$ . The ball has a diameter very slightly less than  $d$ . All surfaces are frictionless. The ball given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by  $\theta$  (shown in figure)
- (a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle  $\theta$ .
- (b) Let  $N_A$  and  $N_B$  denote the magnitudes of the normal reaction force on the ball exerted by the spheres A and B, respectively. Sketch the variations of  $N_A$  and  $N_B$  as functions of  $\cos\theta$  in the range  $0 \leq \theta \leq \pi$  by drawing two separate graphs in your answer book, taking  $\cos\theta$  on the horizontal axis



15. A double star system consists of two stars A and B which have time period  $T_A$  and  $T_B$ . Radius  $R_A$  and  $R_B$  and mass  $M_A$  and  $M_B$ . Choose the correct option. [IIT 2006, (3, -1)]
- (A) If  $T_A > T_B$  then  $R_A > R_B$  (B) If  $T_A > T_B$  then  $M_A > M_B$
- (C)  $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$  (D)  $T_A = T_B$

# ANSWER

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## EXERCISE # 1

### SECTION (A) : KINEMATICS OF CIRCULAR MOTION

A 1. (b) first (b) third.

### SECTION (B) : RADIAL AND TANGENTIAL ACCELERATION

B 1. 2.7 m/sec<sup>2</sup>

### SECTION (C) : RADIUS OF CURVATURE AND DYNAMICS OF CIRCULAR MOTION

C 1.  $8T_0$                       C 2.  $90^\circ$   
 C 3.  $\frac{u^2 \cos^2 \theta}{g \cos^3(\theta/2)}$               C 4. [14.8N, 14.8 N]

### SECTION (D) : CIRCULAR MOTION IN HORIZONTAL PLANE

D 1. 20 rad/s                      D 2. 4.5 mg  
 D 3. 2 m/s                        D 4. 0.40

### SECTION (E) : CIRCULAR MOTION IN VERTICAL PLANE

E 1. (i)  $h_{\min} = \frac{5}{2}r$       (ii)  $F = 6mg$       E 2. 0.8 m  
 E 3. 4 m/sec

### SECTION (F) : MOTION OF A VEHICLE, CENTRIFUGAL FORCE AND ROTATION OF EARTH

F 1. 4.5 cm                      F 2.  $v = \omega \sqrt{r^2 - R^2}$   
 F 3.  $\tan^{-1}(1/4)$               F 4. 0.25  
 F 5. (A)  $3.5 \times 10^{-3}$ , (B) 2.0 hour  
 F 6. Between 14.7 km/h and 54 km/hr]

## EXERCISE # 2

### SECTION (A) : KINEMATICS OF CIRCULAR MOTION

A 1. D                      A 2. C                      A 3. D  
 A 4. AD                      A 5. D                      A 6. BD

### SECTION (B) : RADIAL AND TANGENTIAL ACCELERATION

B 1. C                      B 2. D                      B 3. D  
 B 4. D                      B 5. C                      B 6. A  
 B 7. B                      B 8. D                      B 9. C

### SECTION (C) : RADIUS OF CURVATURE AND DYNAMICS OF CIRCULAR MOTION

C 1. D                      C 2. A

### SECTION (D) : CIRCULAR MOTION IN HORIZONTAL PLANE

D 1. D                      D 2. D                      D 3. A  
 D 4. A                      D 5. A

### SECTION (E) : CIRCULAR MOTION IN VERTICAL

### PLANE

E 1. C                      E 2. A                      E 3. B  
 E 4. D                      E 5. B                      E 6. C  
 E 7. C                      E 8. C                      E 9. D

### SECTION (F) : MOTION OF A VEHICLE, CENTRIFUGAL FORCE AND ROTATION OF EARTH

F 1. A                      F 2. C                      F 3. D  
 F 4. A                      F 5. BC                      F 6. BD  
 F 7. BD

## EXERCISE # 3

1. B                      2. A                      3. C  
 4. D                      5. B                      6. D  
 7. A                      8. B  
 9. (i) A                      (ii) C                      (iii) A  
 10. B  
 11. (i) A                      (ii) B  
 12. (i) A                      (ii) A  
 13. D                      14. A

## EXERCISE # 4

1. A  
 2. (a) 21.65 m/s<sup>2</sup>                      (b) 7.35 m/s                      (c) 12.5 m/s<sup>2</sup>  
 3.  $\frac{5\pi R}{6v}$  ,  $\frac{11\pi}{6}$  ,  $\frac{17v}{5R}$  ,  $\frac{289v^2}{25R}$   
 4. (a)  $v_{\max} = \sqrt{Rg}$  ,  
 (b) At angle  $\theta = 60^\circ$  from the vertical position.  
 (c)  $v = \sqrt{gR \cos\left(\frac{L}{2R}\right)}$

5. (a) 39.6 per min ., (b) 150 N  
 6.  $T = m\ell n^2$

7.  $\sqrt{gr \tan \theta}$  ,  $2\pi \sqrt{\frac{r}{g \tan \theta}}$

8. Net force on car = frictional force f

$$\therefore f = m \sqrt{a^2 + \frac{v^4}{R^2}} \quad (\text{where } m \text{ is mass of the car}) \quad \dots\dots(1)$$

For skidding to just occur

$$f = \mu N = \mu mg \quad \dots\dots(2)$$

$\therefore$  From (1) and (2)

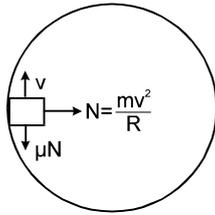
$$v = \{R^2[\mu^2 g^2 - a^2]\}^{1/4}$$

$$\left[ (\mu^2 g^2 - a^2) R^2 \right]^{1/4}$$

9. (A) 975N, 1025 N ,                      (B) 0,707N, 0 ,  
 (C) 682N , 732 N ,                      (D) 0 1.037]

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**EXERCISE # 5**



10. (B)

- (i) The normal reaction by wall on the block is  $N = \frac{mv^2}{R}$
- (ii) The friction force on the block by the wall is  $f = \mu N = \frac{\mu mv^2}{R}$
- (iii) The tangential acceleration of the block =  $\frac{f}{m} = \frac{\mu v^2}{R}$

(iv)  $\frac{dv}{dt} = -\frac{\mu v^2}{R}$

or  $v \frac{dv}{ds} = -\frac{\mu v^2}{R} \Rightarrow \int \frac{dv}{v} = -\frac{\mu}{R} \int ds$

$= -\int_0^{2\pi R} \frac{\mu}{R} ds$

integrating we get

$\ln \frac{v}{v_0} = -\mu \cdot 2\pi$

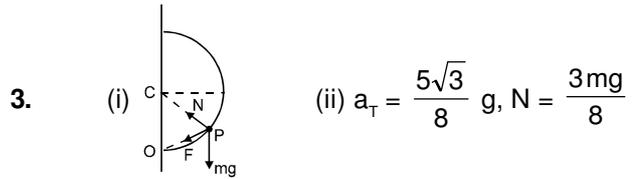
$v = v_0 e^{-2\mu\pi}$

Ans. (B)

(i)  $\frac{mv^2}{R}$  (ii)  $\frac{\mu mv^2}{R}$

(iii)  $-\frac{\mu v^2}{R}$  (iv)  $v_0 e^{-2\mu\pi}$

1. (a)  $7\sqrt{2}$  rad/s (b)  $-9.8 \times 10^{-3}$  m/s<sup>2</sup>
2. B



4. (i) 36 N (ii) 11.67 rad/s  
(iii)  $r_1 = 0.1$  m and  $r_2 = 0.2$  m

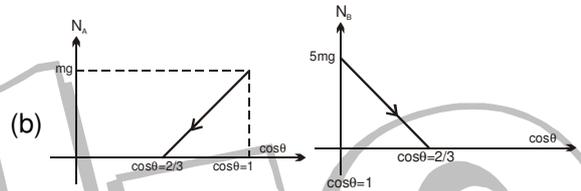
5.  $(\sqrt{65})mg, 3R$  6. D

7.  $H = (5/3)R$  8.  $u = \sqrt{gL \left( \frac{3\sqrt{3}}{2} + 2 \right)}$

9. A 10. A 11. A

12. 0.24m, 6.4rad/sec 13. C

14. (a)  $N = 3mg \cos\theta - 2mg$



15. D

11.  $\sqrt{\frac{2L}{\omega^2 R \cos\theta}}$

12. (A) 0.2N, (B) 30°

13.  $\frac{\omega^2 R}{3} \cdot \frac{4}{3} m \omega^2 R$